Analytical Asymmetry Parameters for Symmetrical Waveguide Junctions*

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Summary—**This paper presents a systematic approach to the evaluation of (waveguide) junctions from the standpoint of their conformance to certain symmetries. Preferred sets of asymmetry parameters are found which are complete, minimal in number, which go to zero when the junction represented is symmetrical, and which may often be identified with a corresponding structural symmetry defect. The asymmetry parameters are first introduced for general linear junctions, but special attention is given to reciprocal and lossless junctions. The derivation of these preferred sets is based on the theory of group representations hitherto employed in the analysis of ideally symmetric junctions. One of the applications of these preferred parameters yields first-order relations among the defects of a nearly perfect hybrid-T junction which are believed to be new.**

I. Introduction

THIS PAPER presents a systematic approach to the evaluation of waveguide junctions from the standpoint of their conformance to certain symmetries. While ideal symmetrical junctions have received extensive treatment in the recent literature,[1–6] little account is taken, in these papers, of the fact that all actual junctions are, in some degree, asymmetrical. On specialized consideration of a particular junction, engineers have commonly improvised parameters descriptive of junction asymmetries. It generally remained uncertain whether or not such a set of asymmetry parameters, introduced *ad hoc*, was either complete (in the sense that any arbitrary asymmetry could be described) or minimal (in the sense that no linear relations subsisted among elements of the set). Here, these questions are resolved simply and, we believe, naturally in terms of the same theoretical framework which has been successfully employed in the analysis of ideal symmetrical junctions; *i.e.*, the theory of linear transformations and representations of finite point groups.

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[1] B. A. Auld, "Applications of Group Theory in the Study of Symmetrical Waveguide Junctions," Stanford University, Stanford, Calif., MLR-157; March, 1952.

[2] H. A. Folwer, "Reciprocity and the Scattering Matrix of Ferrite Devices," Electronics Res. Lab., Brown University, Providence, R. I.

[3] W. K. Kahn, "Scattering equivalent circuits for common symmetrical junctions," IRE Trans. on Circuit Theory, vol. CT-3, pp. 121–127; June, 1956.

[4] D. M. Kerns, "Analysis of symmetrical waveguide junctions," *J. Res., NBS*, vol. 46, pp. 267–282; April, 1951.

[5] C. G. Montgommery, R. H. Dicke, and E. M. Purcell, "Principles of Microwave Circuits," McGraw-Hill Book Co., Inc., New York, N. Y.; 1948.

[6] A. E. Pannenborg, "On the scattering matrix of symmetrical waveguide junctions," *Phillips Res. Repts.*, vol. 7, pp. 133; April, 1952.

A procedure is outlined whereby a *preferred set of asymmetry parameters* may be derived for any junction appropriate for description of the degree in which that junction deviates from a given symmetry. The parameters comprised in such a set are complete, minimal in number, and all go to zero if, and only if, the junction is symmetrical (or electrically equivalent to a junction with the required symmetry). They are termed analytical asymmetry parameters because particular structural symmetry defects may often be deduced from them. First obtained for general linear junctions, special attention is given to reciprocal and lossless junctions. Scattering notation has been employed throughout this paper for the network quantities since these are the most convenient for microwave junctions, and moreover they exist for arbitrary passive structures.

The principles by means of which the analytical asymmetry parameters may be derived is sketched in Section II. This sketch may largely be supplemented by reference to the extensive treatments of ideal symmetrical junctions previously cited. A detailed illustration of the procedure is presented in Section III, in which asymmetry parameters appropriate to the H-plane Y junction are derived. This section should also clarify the special case of symmetry degeneracy, which is slighted in Section II for the sake of brevity.

The final section contains two examples illustrating the measurement and theoretical significance of the derived asymmetry parameters. The results of the perturbation calculation performed on a nearly perfect hybrid-T are believed to be new.

II. Symmetry and Asymmetry Parameters

At any frequency the network characteristics of a linear N-port, equivalent to a particular junction (one without "noncontrolled" sources) at reference planes appropriately chosen in perfectly conducting uniform waveguide leads, may be described by N^2 complex parameters. The elements of the conventional (normalized, voltage) scattering matrix,

$$S = (\mathcal{S}_{ij}) \qquad i, j = 1, 2, \cdots, N, \tag{1}$$

constitute one such description. This matrix relates the column matrices of terminal quantities $\boldsymbol{a}$ and $\boldsymbol{b}$;

$$\boldsymbol{b} = S\boldsymbol{a}, \tag{2}$$

the elements of which,

$$\boldsymbol{a} = (a_i) \quad \text{and} \quad \boldsymbol{b} = (b_i) \qquad i = 1, 2, \cdots, N, \tag{3}$$

are, respectively, the rms phasors corresponding to the waves incident onto and reflected from the junction at the reference planes chosen. These phasors are so normalized that a^+a and b^+b are, respectively, equal to the power incident onto and the power reflected from the junction. (The symbol a^+ denotes the conjugate transpose of a.) In this section, alternative (scattering) descriptions will be developed, entirely equivalent in point of generality to the conventional scattering matrix, but especially appropriate to junctions conforming to particular symmetries. The N^2 complex parameters entering into such a description fall into one of two categories: 1) those parameters which *necessarily vanish* when the junction represented actually conforms to the particular symmetries which determined the description; and 2) the parameters which do *not necessarily vanish* in that case. Those in the first category are denoted *asymmetry parameters*, and those in the second, *symmetry parameters*. The elements of the conventional scattering matrix will be expressed (linearly) in terms of the asymmetry and symmetry parameters, and conversely.

Consider a symmetrical structure such as, for example, the waveguide junction shown in Fig. 1. The physical symmetry of such a structure may be described in terms of the operations, *i.e.*, reflections and rotations, which leave the structure invariant. These operations form one representation of a group, the symmetry or point group of the junction. The corresponding electrical symmetry of the network equivalent to the junction may be described in terms of the permutations of the terminal quantities which leave the network relation (2) invariant. The unitary matrices M_k, which perform these permutations of the terminal quantities a and b, may be written down by inspection, and these then form another representation of the mentioned group. Thus, if

$$b = Sa \tag{4}$$

and

$$a^{(k)} = M_k a, \qquad b^{(k)} = M_k b = M_k S a, \tag{5}$$

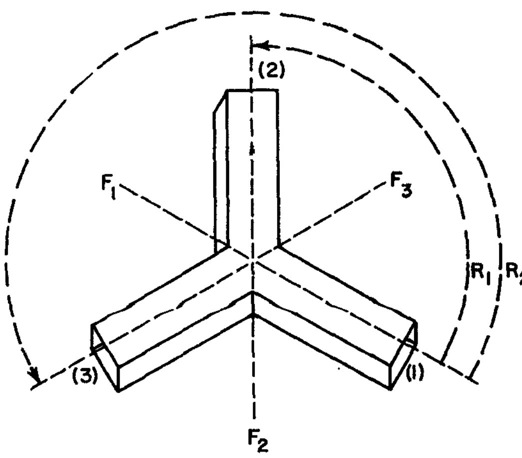


Fig.1—Symmetrical waveguide junction, *H*-plane *Y*. (Symmetry planes marked *F*, rotational symmetries marked *R*.)

then

$$b^{(k)} = Sa^{(k)} \tag{6}$$

for *arbitrary* a. When $a^{(k)}$ and $b^{(k)}$ in (6) are replaced by their equivalents in terms of a, there results:

$$M_k S a = S M_k a, \tag{7}$$

which yields the essential connection between M_k and S,

$$M_k S = S M_k. \tag{8}$$

This connection may be utilized directly to find the relations among the conventional scattering coefficients that are a result of the symmetry to which M_k corresponds [Section III, (27)–(29)]. More to the purpose at hand, (8) may also be utilized to find a transformation which reduces the scattering matrix of a symmetrical junction.

Since S and M_k commute, it is known that these two matrices have a common set of invariant subspaces.[7] But the permutation matrix M_k is simple in form and is known (having been deduced from the geometrical symmetry of the junction). Hence, invariant subspaces of S will be found by finding the unique invariant subspaces of M_k, and from these a transformation will be constructed which reduces S.

The eigenvectors $m_k^{(i)}$ belonging to the eigenvalues $\mu_k^{(i)}$ of M_k satisfy the relation

$$(M_k - \mu_k^{(i)}) m_k^{(i)} = 0, \qquad i = 1, 2, \cdots. \tag{9}$$

N linearly independent eigenvectors may be arranged as a hermitean orthonormal set since M_k is unitary. Assign consecutive superscripts to any repeated degenerate eigenvalues. The subspaces spanned by all the eigenvectors corresponding to any one value are the unique invariant subspaces of M_k. Then the transformation,

$$T_k = (m_k^{(1)} m_k^{(2)} \cdots m_k^{(N)}), \tag{10}$$

formed with these eigenvectors as columns, is unitary; *i.e.*, $T^{-1} = T^+$. Acting on columns a_k and b_k,

$$a = T_k a_k, \qquad b = T_k b_k; \tag{11}$$

T_k expresses a and b as linear combinations of the eigenvectors $m_k^{(i)}$. The column matrices a_k and b_k may be regarded as new or transformed (incident and reflected wave) terminal quantities. The transformed scattering matrix S_k corresponding to these new terminal quantities may be found on substitution for a and b in (2).

$$T_k b_k = S T_k a_k, \tag{12}$$

$$b_k = T_k^{-1} S T_k a_k = T_k^+ S T_k a_k, \tag{13}$$

and comparing this result with the defining equation;

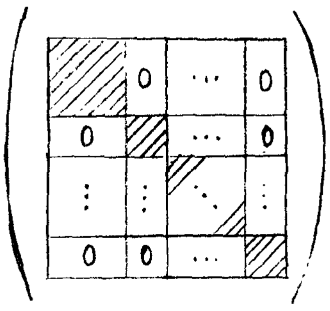
$$b_k = S_k a_k; \tag{14}$$

[7] H. L. Hamburger and M. E. Grimshaw, "Linear Transformations in *N*-Dimensional Vector Space," Cambridge University Press, Cambridge, England, ch. 23, p. 138; 1956.

i.e.,

$$S_k = T_k^+ S T_k. \tag{15}$$

The matrix S_k has the form

$S_k =$  , (16)

where the shaded regions along the principal diagonal of S_k represent square submatrices. The elements of these submatrices will be denoted Q_{ij}. Each submatrix corresponds to an eigenvalue μ_k of M_k; the dimension of the submatrix is equal to the degeneracy of that eigenvalue. The zeros in the remaining rectangles imply that the elements in these submatrices of S_k are *all necessarily zero.*

To recapitulate: if a junction actually possesses the symmetry corresponding to M_k, then its scattering matrix S commutes with M_k and the matrix S_k defined in (15) necessarily has the form (16). The elements Q_{ij} then suffice to describe the junction.

Now consider an *arbitrary* waveguide junction. Its scattering matrix S does not (necessarily) commute with M_k. If, nevertheless, S_k is defined by (15), S_k is entirely general in form with no elements (necessarily) equal to zero. Retain the notation Q_{ij} for those elements with subscripts ij for which it was introduced in the symmetrical case, and denote the remaining elements of S_k q_{ij}. Then the q_{ij} are precisely those scattering parameters which necessarily vanish when the junction represented actually conforms to the particular symmetry corresponding to M_k; *i.e.*, the asymmetry parameters. The Q_{ij} are the corresponding symmetry parameters.

The above theory may readily be extended to include more complex symmetries to which several or a whole group of matrices M_k, $k=1, 2, \cdots$ correspond. An example of the procedure may be found in Section III.

It was suggested in connection with (11)–(15) that the matrix S_k be regarded as an alternative or transformed scattering description with terminal quantities $\boldsymbol{a}_k$ and $\boldsymbol{b}_k$. One way in which this viewpoint may be made useful and, perhaps, more familiar, is by displaying the special forms that this matrix takes when the junction represented is nondissipative and Lorentz reciprocal; a second way is presented in the last section.

When a junction is nondissipative, the conventional scattering matrix S, descriptive of this junction, is unitary. But,

$$S_k^{-1} = (T_k^+ S T_k)^{-1} = T_k^{-1} S^{-1} (T_k^+)^{-1} = T_k^+ S^{-1} T_k, \tag{17}$$

$$S_k^+ = (T_k^+ S T_k)^+ = T_k^+ S^+ T_k, \tag{18}$$

since T_k is unitary; hence, when S is unitary

$$S_k^{-1} = S_k^+, \tag{19}$$

or S_k is also unitary.

When a junction is Lorentz reciprocal, the conventional scattering matrix descriptive of this junction has

$$S = \tilde{S}. \tag{20}$$

($\tilde{S}$ denotes the transpose matrix of S.) Substituting for S its expression in terms of S_k:

$$T_k S_k T_k^+ = \widetilde{T_k S_k T_k^+}, \tag{21}$$

or

$$(\tilde{T}_k T_k) S_k = \tilde{S}_k (\tilde{T}_k T_k). \tag{22}$$

When, in addition to being unitary, the transformation T_k is real, then

$$T_k^{-1} = \tilde{T}_k \tag{23}$$

and (22) reduces to

$$S_k = \tilde{S}_k, \tag{24}$$

i.e., the same formal condition on S_k as was imposed on S.

The general theory of this section separates the N^2 independent parameters descriptive of a linear junction into symmetry and asymmetry parameters. Stipulations in addition to linearity regarding the physical character of the junction, such as reciprocity and the conservation of energy force relations among these parameters or, alternatively phrased, reduce the number of parameters which may be assigned arbitrarily. In the instance of reciprocity, $N(N-1)/2$ linear constraints [(20), (22) or (24)] result, and for all the junctions treated, these are so simple that no difficulty is encountered in selecting $N^2 - N(N-1)/2 = (N+1)N/2$ independent parameters. The nonlinear constraints (19) which result from the conservation of energy are not automatically satisfied by the parameters. An illustration of how these nonlinear constraints may be employed is given in the last section.

III. Illustrative Example

While the general principles by means of which appropriate symmetry and asymmetry parameters may be introduced for any junction were presented in the preceding section, these will now be made concrete by application to the H-plane Y junction shown in Fig. 1. This junction constitutes the simplest example which displays all the idiosyncrasies encountered in the most general case.

The symmetry operations have been indicated by marking the planes of reflection symmetry F_1, F_2, F_3, respectively, and the 120°, 240° rotations by R_1 and R_2, respectively.[8] The unitary matrices which perform per-

[8] Subsequently, the same notation will be employed for the geometrical symmetry, the symmetry operation which leaves the junction invariant, and the corresponding permutation matrix.

mutations of the terminal quantities corresponding to F_1 and R_1 are

$$F_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \qquad R_1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \tag{25}$$

Note that $F_1^2 = I$ and $R_1^3 = I$.

The remaining matrices may be found from the two given by matrix multiplication in accordance with the multiplication Table A below.

TABLE A

M_i \ M_j	I	R_1	R_2	F_1	F_2	F_3
I	I	R_1	R^2	F_1	F_2	F_3
R_1	R_1	R_2	I	F_2	F_3	F_1
R_2	R_2	I	R_1	F_3	F_1	F_2
F_1	F_1	F_3	F_2	I	R_2	R_1
F_2	F_2	F_1	F_3	R_1	I	R_2
F_3	F_3	F_2	F_1	R_2	R_1	I

This table is to be read:

$$M_j M_i = M_k, \tag{26}$$

where

M_i = the ith element of the first column,
M_j = the jth element of the first row,
M_k = the element at the intersection of the ith row and jth column.

Each entry in Table A may be verified by reference to Fig. 1, where the effect of operation M_i *followed* by operation M_j may be seen geometrically.

In the frequency range in which only one mode propagates in each of the waveguide leads, the scattering matrix S of the Y junction with respect to symmetrically chosen reference planes, may be written:

$$S = \begin{bmatrix} \mathcal{S}_{11} & \mathcal{S}_{12} & \mathcal{S}_{13} \\ \mathcal{S}_{21} & \mathcal{S}_{22} & \mathcal{S}_{23} \\ \mathcal{S}_{31} & \mathcal{S}_{32} & \mathcal{S}_{33} \end{bmatrix}. \tag{27}$$

If the Y Junction truly conforms to the symmetry F_1, we have from (8):

$$F_1 S = S F_1, \tag{28}$$

which, on multiplying out, is seen to imply:

$$\mathcal{S}_{12} = \mathcal{S}_{13}, \quad \mathcal{S}_{21} = \mathcal{S}_{31}, \quad \mathcal{S}_{22} = \mathcal{S}_{33}, \text{ and } \mathcal{S}_{23} = \mathcal{S}_{32}. \tag{29}$$

In order to find the symmetry and asymmetry parameters appropriate to F_1, the transformation T_{F_1} must be constructed from eigenvectors $f_1^{(i)}$ of F_1. Accordingly, consider the eigenvalue problem:

$$(F_1 - \phi_1^{(i)}) f_1^{(i)} = 0. \tag{30}$$

The eigenvalues $\phi_1^{(i)}$ are found as the roots of

$$\det (F_1 - \phi_1 I) = 0 = (\phi_1 - 1)(\phi_1^2 - 1), \tag{31}$$

or

$$\phi_1^{(1)} = +1, \quad \phi_1^{(2)} = +1, \text{ and } \phi_1^{(3)} = -1.$$

Since $\phi_1^{(1)} = \phi_1^{(2)}$, the eigenvalue problem is degenerate; *i.e.*, the invariant subspace belonging to the eigenvalue $+1$ is two-dimensional. Many pairs of eigenvectors which span the subspace belonging to the eigenvalue $+1$ may be found. Perhaps the simplest orthonormal set is that given in Table B.

TABLE B

Eigenvalue	$\phi_1^{(1)} = \phi_1^{(2)} = +1$		$\phi_1^{(3)} = -1$
Corresponding eigenvector(s)	$f_1^{(1)} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$	$f_1^{(2)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$	$f_1^{(3)} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

The transformation T_{F_1} constructed from these eigenvectors is

$$T_{F_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}. \tag{32}$$

In accordance with (16), the matrix $S_{F_1} = T_{F_1}^{+} S T_{F_1}$ has the form

$$S_{F_1} = \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} & 0 \\ \mathcal{A}_{21} & \mathcal{A}_{22} & 0 \\ 0 & 0 & \mathcal{A}_{33} \end{bmatrix}, \tag{33}$$

provided that the Y Junction truly conforms to the symmetry F_1. Therefore, in general, $S_{F_1} = T_{F_1}^{+} S T_{F_1}$ is given by

$$S_{F_1} = \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} & a_{13} \\ \mathcal{A}_{21} & \mathcal{A}_{22} & a_{23} \\ a_{31} & a_{32} & \mathcal{A}_{33} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \mathcal{S}_{22} + \mathcal{S}_{32} + \mathcal{S}_{23} + \mathcal{S}_{32} & \sqrt{2}(\mathcal{S}_{21} + \mathcal{S}_{31}) & \mathcal{S}_{22} - \mathcal{S}_{33} + \mathcal{S}_{32} - \mathcal{S}_{23} \\ \sqrt{2}(\mathcal{S}_{12} + \mathcal{S}_{13}) & 2\mathcal{S}_{11} & \sqrt{2}(\mathcal{S}_{12} - \mathcal{S}_{13}) \\ \mathcal{S}_{22} - \mathcal{S}_{33} + \mathcal{S}_{23} - \mathcal{S}_{32} & \sqrt{2}(\mathcal{S}_{21} - \mathcal{S}_{31}) & \mathcal{S}_{22} + \mathcal{S}_{33} - \mathcal{S}_{23} - \mathcal{S}_{32} \end{bmatrix}, \tag{34}$$

where the upper case $\mathcal{A}_{ij}$ are symmetry and the lower case a_{ij} asymmetry, parameters. Inversely, $S = T_{F_1} S_{F_1} T_{F_1}^{+}$ is given by

$$S = \begin{bmatrix} \mathcal{S}_{11} & \mathcal{S}_{12} & \mathcal{S}_{13} \\ \mathcal{S}_{12} & \mathcal{S}_{22} & \mathcal{S}_{23} \\ \mathcal{S}_{31} & \mathcal{S}_{32} & \mathcal{S}_{33} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2\mathcal{A}_{22} & \sqrt{2}(\mathcal{A}_{21} - a_{23}) & \sqrt{2}(\mathcal{A}_{21} - a_{23}) \\ \sqrt{2}(\mathcal{A}_{13} + a_{32}) & \mathcal{A}_{11} + \mathcal{A}_{33} + a_{13} + a_{31} & \mathcal{A}_{11} - \mathcal{A}_{33} - a_{13} + a_{31} \\ \sqrt{2}(\mathcal{A}_{12} - a_{32}) & \mathcal{A}_{11} - \mathcal{A}_{33} + a_{13} - a_{31} & \mathcal{A}_{11} + \mathcal{A}_{22} - a_{13} - a_{31} \end{bmatrix}. \tag{35}$$

These results are also listed in Table I(a).

Returning to the eigenvalue problem, (30), an alternative set of orthonormal eigenvectors which will prove useful subsequently is given in Table C.

TABLE C

Eigenvalues	$\phi_1^{(1)} = \phi_1^{(2)} = +1$		$\phi_1^{(3)} = -1$
Corresponding eigenvector(s)	$\hat{f}_1^{(1)} = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix}$	$\hat{f}_1^{(2)} = \frac{1}{\sqrt{6}}\begin{bmatrix}2\\-1\\-1\end{bmatrix}$	$\hat{f}_1^{(3)} = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\-1\end{bmatrix}$

The transformation $\hat{T}_{F_1}$ now appears as

$$\hat{T}_{F_1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix}. \tag{36}$$

Corresponding (but different) symmetry and asymmetry parameters may be introduced to parallel (33)–(35).

In order to find the parameters appropriate to R_1, the transformation T_{R_1} must be constructed from the eigenvectors $r_1^{(i)}$ of R_1. Accordingly, consider the eigenvalue problem

$$(R_1 - \rho_1^{(i)})r_1^{(i)} = 0. \tag{37}$$

The eigenvalues $\rho_1^{(i)}$ are found as the roots of

$$\det\,(R_1 - \rho_1 I) = 0 = (\rho_1^3 - 1),$$

or

$$\rho_1^{(1)} = 1, \qquad \rho_1^{(2)} = k_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2j},$$

and

$$\rho_1^{(3)} = k_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2j}.$$

Since the three roots are distinct, the normalized eigenvectors are uniquely those given in Table D.

TABLE D

Eigenvalue	$\rho_1^{(1)} = 1$	$\rho_1^{(2)} = k_1$	$\rho_1^{(3)} = k_2$
Corresponding eigenvector	$r_1^{(1)} = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix}$	$r_1^{(2)} = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\k_2\\k_1\end{bmatrix}$	$r_1^{(3)} = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\k_1\\k_2\end{bmatrix}$

The transformation T_{R_1} constructed from these eigenvectors is

$$T_{R_1} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & k_2 & k_1 \\ 1 & k_1 & k_2 \end{bmatrix}. \tag{38}$$

In accordance with (16), the transformed scattering matrix $S_{R_1} = T_{R_1}^{+} S T_{R_1}$ has the form

$$S_{R_1} = \begin{bmatrix} D_{11} & 0 & 0 \\ 0 & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix}, \tag{39}$$

provided that the Y junction truly conforms to the symmetry R_1. The general expressions for S_{R_1} and S in terms of symmetry and asymmetry parameters are listed in Table I(b).

When symmetries F_1 and R_1 obtain simultaneously, then the junction is perfectly symmetrical; *i.e.*, when

$$F_1 S = S F_1 \quad \text{and} \quad R_1 S = S R_1, \tag{40}$$

then similar relations hold for F_2, F_3, and R_2, for these may be expressed in terms of F_1 and R_1; *cf.*, Table A. In order to find the parameters appropriate to this symmetry, the transformation $T_{F_1 \& R_1}$ must be constructed. Both eigenvalue problems (30) and (37) are in point here since the scattering matrix of a perfectly symmetrical junction, by (40), must have a set of eigenvectors in common with each F_1 and R_1. Comparison of the eigenvectors in Tables C and D shows that while the first eigenvectors of F_1 and R_1 listed there agree, the remaining two do not. Hence, the requirements imposed by (40) on the eigenvectors of the scattering matrix of a *perfectly symmetrical junction* may be satisfied only if the eigenvalue problem

$$(S - \sigma^{(i)} I) s^{(i)} = 0 \tag{41}$$

is degenerate. That the vectors $\hat{f}_1^{(2)}$, $\hat{f}_1^{(3)}$ and $r_1^{(2)}$, $r_1^{(3)}$ span the same subspace follows from their orthogonality (to $\hat{f}_1^{(1)} = r_1^{(1)}$). Hence, if the eigenvector $s_1^{(1)} = \hat{f}_1^{(1)} = r_1^{(1)}$ corresponds to $\sigma^{(1)}$ then

$$\sigma^{(2)} = \sigma^{(3)} \tag{42}$$

is a necessary and sufficient condition on S to satisfy (40). The eigenvectors corresponding to $\sigma^{(2)} = \sigma^{(3)}$ may then be chosen as $\hat{f}_1^{(2)}$, $\hat{f}_1^{(3)}$; $r_1^{(2)}$, $r_1^{(3)}$; or any other linear combination of these. Selecting the first of these alternatives, it follows that

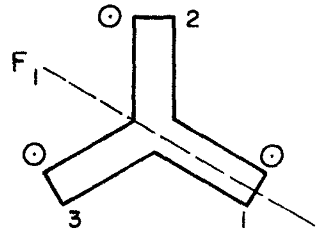
$$T_{F_1 \& R_1} = \hat{T}_{F_1},$$

and that for a perfectly symmetrical junction, $S_{F_1 \& R_1} = T_{F_1 \& R_1}^{+} S T_{F_1 \& R_1}$ has the form

TABLE I

(a) SYMMETRICAL NONRECIPROCAL H-PLANE Y JUNCTION F_1 SYMMETRY PLANE

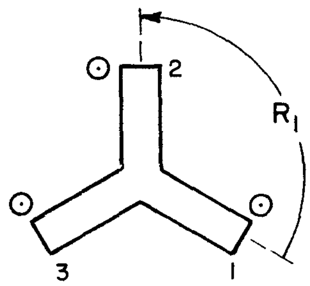
$$F_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad T_{F_1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \\ 1 & 0 & -1 \end{pmatrix}$$



Natural Basis	$S = \begin{pmatrix} \mathcal{S}_{11} & \mathcal{S}_{12} & \mathcal{S}_{13} \\ \mathcal{S}_{21} & \mathcal{S}_{22} & \mathcal{S}_{23} \\ \mathcal{S}_{31} & \mathcal{S}_{32} & \mathcal{S}_{33} \end{pmatrix}$	$S = \frac{1}{2} \begin{pmatrix} 2\mathcal{A}_{22} & \sqrt{2}(\mathcal{A}_{21} + a_{23}) & \sqrt{2}(\mathcal{A}_{21} - a_{23}) \\ \sqrt{2}(\mathcal{A}_{12} + a_{32}) & (\mathcal{A}_{11} + \mathcal{A}_{33} + a_{13} + a_{31}) & (\mathcal{A}_{11} - \mathcal{A}_{33} - a_{13} + a_{31}) \\ \sqrt{2}(\mathcal{A}_{12} - a_{32}) & (\mathcal{A}_{11} - \mathcal{A}_{33} + a_{13} - a_{31}) & (\mathcal{A}_{11} + \mathcal{A}_{33} - a_{13} - a_{31}) \end{pmatrix}$
Transformed Basis	$S_{F_1} = \begin{pmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} & a_{13} \\ \mathcal{A}_{21} & \mathcal{A}_{22} & a_{23} \\ a_{31} & a_{32} & \mathcal{A}_{33} \end{pmatrix}$	$S_{F_1} = \frac{1}{2} \begin{pmatrix} (\mathcal{S}_{22} + \mathcal{S}_{33} + \mathcal{S}_{23} + \mathcal{S}_{32}) & \sqrt{2}(\mathcal{S}_{21} + \mathcal{S}_{31}) & (\mathcal{S}_{22} - \mathcal{S}_{33} + \mathcal{S}_{32} - \mathcal{S}_{23}) \\ \sqrt{2}(\mathcal{S}_{12} + \mathcal{S}_{13}) & 2\mathcal{S}_{11} & \sqrt{2}(\mathcal{S}_{12} - \mathcal{S}_{13}) \\ (\mathcal{S}_{22} - \mathcal{S}_{33} + \mathcal{S}_{23} - \mathcal{S}_{32}) & \sqrt{2}(\mathcal{S}_{21} - \mathcal{S}_{31}) & (\mathcal{S}_{22} + \mathcal{S}_{33} - \mathcal{S}_{23} - \mathcal{S}_{32}) \end{pmatrix}$

(b) SYMMETRICAL NONRECIPROCAL H-PLANE Y JUNCTION R_1 ROTATIONAL SYMMETRY

$$R_1 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad T_{R_1} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & k_2 & k_1 \\ 1 & k_1 & k_2 \end{pmatrix}$$



Natural Basis	$S = \begin{pmatrix} \mathcal{S}_{11} & \mathcal{S}_{12} & \mathcal{S}_{13} \\ \mathcal{S}_{21} & \mathcal{S}_{22} & \mathcal{S}_{23} \\ \mathcal{S}_{31} & \mathcal{S}_{32} & \mathcal{S}_{33} \end{pmatrix}$	$\mathcal{S}_{11} = \frac{1}{3}[\mathcal{D}_{11} + \mathcal{D}_{22} + \mathcal{D}_{33} + d_{12} + d_{13} + d_{21} + d_{23} + d_{31} + d_{32}]$ $\mathcal{S}_{12} = \frac{1}{3}[\mathcal{D}_{11} + \mathcal{D}_{22}k_1 + \mathcal{D}_{33}k_2 + k_1(d_{12} + d_{32}) + k_2(d_{13} + d_{23}) + d_{21} + d_{31}]$ $\mathcal{S}_{13} = \frac{1}{3}[\mathcal{D}_{11} + \mathcal{D}_{22}k_2 + \mathcal{D}_{33}k_1 + k_1(d_{13} + d_{23}) + k_2(d_{12} + d_{32}) + d_{21} + d_{31}]$ $\mathcal{S}_{21} = \frac{1}{3}[\mathcal{D}_{11} + \mathcal{D}_{22}k_2 + \mathcal{D}_{33}k_1 + k_1(d_{31} + d_{32}) + k_2(d_{21} + d_{23}) + d_{12} + d_{13}]$ $\mathcal{S}_{22} = \frac{1}{3}[\mathcal{D}_{11} + \mathcal{D}_{22} + \mathcal{D}_{33} + k_1(d_{12} + d_{23} + d_{31}) + k_2(d_{13} + d_{21} + d_{32})]$ $\mathcal{S}_{23} = \frac{1}{3}[\mathcal{D}_{11} + \mathcal{D}_{22}k_1 + \mathcal{D}_{33}k_2 + k_1(d_{13} + d_{21} + d_{32}) + k_2(d_{12} + d_{23} + d_{31})]$ $\mathcal{S}_{31} = \frac{1}{3}[\mathcal{D}_{11} + \mathcal{D}_{22}k_1 + \mathcal{D}_{33}k_2 + k_1(d_{21} + d_{23}) + k_2(d_{31} + d_{32}) + d_{12} + d_{13}]$ $\mathcal{S}_{32} = \frac{1}{3}[\mathcal{D}_{11} + \mathcal{D}_{22}k_2 + \mathcal{D}_{33}k_1 + k_1(d_{12} + d_{21}) + k_2(d_{13} + d_{31}) + d_{23} + d_{32}]$ $\mathcal{S}_{33} = \frac{1}{3}[\mathcal{D}_{11} + \mathcal{D}_{22} + \mathcal{D}_{33} + k_1(d_{13} + d_{21} + d_{32}) + k_2(d_{12} + d_{23} + d_{31})]$
Transformed Basis	$S_{R_1} = \begin{pmatrix} \mathcal{D}_{11} & d_{12} & d_{13} \\ d_{21} & \mathcal{D}_{22} & d_{23} \\ d_{31} & d_{32} & \mathcal{D}_{33} \end{pmatrix}$	$\mathcal{D}_{11} = \frac{1}{3}[\mathcal{S}_{11} + \mathcal{S}_{22} + \mathcal{S}_{33} + \mathcal{S}_{12} + \mathcal{S}_{13} + \mathcal{S}_{21} + \mathcal{S}_{23} + \mathcal{S}_{31} + \mathcal{S}_{32}]$ $d_{12} = \frac{1}{3}[\mathcal{S}_{11} + \mathcal{S}_{22}k_2 + \mathcal{S}_{33}k_1 + k_1(\mathcal{S}_{13} + \mathcal{S}_{23}) + k_2(\mathcal{S}_{12} + \mathcal{S}_{32}) + \mathcal{S}_{21} + \mathcal{S}_{31}]$ $d_{13} = \frac{1}{3}[\mathcal{S}_{11} + \mathcal{S}_{22}k_1 + \mathcal{S}_{33}k_2 + k_1(\mathcal{S}_{12} + \mathcal{S}_{32}) + k_2(\mathcal{S}_{13} + \mathcal{S}_{23}) + \mathcal{S}_{21} + \mathcal{S}_{31}]$ $d_{21} = \frac{1}{3}[\mathcal{S}_{11} + \mathcal{S}_{22}k_1 + \mathcal{S}_{33}k_2 + k_1(\mathcal{S}_{21} + \mathcal{S}_{23}) + k_2(\mathcal{S}_{31} + \mathcal{S}_{32}) + \mathcal{S}_{12} + \mathcal{S}_{13}]$ $\mathcal{D}_{22} = \frac{1}{3}[\mathcal{S}_{11} + \mathcal{S}_{22} + \mathcal{S}_{33} + k_1(\mathcal{S}_{13} + \mathcal{S}_{21} + \mathcal{S}_{32}) + k_2(\mathcal{S}_{12} + \mathcal{S}_{23} + \mathcal{S}_{31})]$ $d_{23} = \frac{1}{3}[\mathcal{S}_{11} + \mathcal{S}_{22}k_2 + \mathcal{S}_{33}k_1 + k_1(\mathcal{S}_{12} + \mathcal{S}_{21}) + k_2(\mathcal{S}_{13} + \mathcal{S}_{31}) + \mathcal{S}_{23} + \mathcal{S}_{32}]$ $d_{31} = \frac{1}{3}[\mathcal{S}_{11} + \mathcal{S}_{22}k_2 + \mathcal{S}_{33}k_1 + k_1(\mathcal{S}_{31} + \mathcal{S}_{32}) + k_2(\mathcal{S}_{21} + \mathcal{S}_{23}) + \mathcal{S}_{12} + \mathcal{S}_{13}]$ $d_{32} = \frac{1}{3}[\mathcal{S}_{11} + \mathcal{S}_{22}k_1 + \mathcal{S}_{33}k_2 + k_1(\mathcal{S}_{13} + \mathcal{S}_{31}) + k_2(\mathcal{S}_{12} + \mathcal{S}_{21}) + \mathcal{S}_{23} + \mathcal{S}_{32}]$ $\mathcal{D}_{33} = \frac{1}{3}[\mathcal{S}_{11} + \mathcal{S}_{22} + \mathcal{S}_{33} + k_1(\mathcal{S}_{12} + \mathcal{S}_{23} + \mathcal{S}_{31}) + k_2(\mathcal{S}_{13} + \mathcal{S}_{21} + \mathcal{S}_{23})]$

$$k_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2j}, \qquad k_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2j}$$

$$S_{F_1\&R_1} = \begin{bmatrix} \mathcal{E}_{11} & 0 & 0 \\ 0 & \mathcal{E}_{22} & 0 \\ 0 & 0 & \mathcal{E}_{22} \end{bmatrix}, \tag{43}$$

where $\sigma^{(1)}$ and $\sigma^{(2)} = \sigma^{(3)}$ have been replaced, respectively, by $\mathcal{E}_{11}$ and $\mathcal{E}_{22}$ to conform to the notation of Section II.

The general expression for $S_{F_1 \& R_1}$ (for *any* three-port) in terms of symmetry and asymmetry parameters takes the form

$$S_{F_1\&R_1} = \begin{bmatrix} \mathcal{E}_{11} & e_{12} & e_{13} \\ e_{21} & \mathcal{E}_{22} + e_{22} & e_{23} \\ e_{31} & e_{32} & \mathcal{E}_{22} - e_{22} \end{bmatrix}. \tag{44}$$

The asymmetry parameter e_{22} is required by the symmetry degeneracy. (More complex combinations of symmetries in junctions with large numbers of ports are more systematically handled by the apparatus of the theory of group representations.)

It is unnecessary to repeat, in each case, for symmetries F_2, F_3 and R_2, discussions equivalent to those just completed for F_1 and R_1. F_1 and R_1 constitute generators of the group, Table A, and hence the parameters for the symmetries F_2 and F_3 may be obtained via, in essence, a relabeling of the ports in Fig. 1. Since R_1 and R_2 commute, the results for S_{R_1} and S_{R_2} are identical. The procedure may be formalized in terms of the symmetry matrices.

Assume that for some symmetry M_k, the eigenvalue problem (9) has been solved; the transformation T_k, (10), has been found, and the form of S_k, (15), determined. From these, it is easy to obtain corresponding results for a matrix M_l.

$$M_k = M_j^{-1} M_l M_j. \tag{45}$$

Substituting for M_k in (9), the expression (45) yields:

$$(M_l - \mu_k^{(i)}) M_j \boldsymbol{m}_k^{(i)} = 0. \tag{46}$$

Thus, the eigenvalues of M_l are precisely those of M_k, namely $\mu_k^{(i)}$, and the corresponding eigenvectors are $M_j \boldsymbol{m}_k^{(i)}$. The transformation T_l is therefore

$$T_l = M_j T_k, \tag{47}$$

and the form of the transformed scattering matrix

$$\begin{aligned} S_l &= T_l^+ S T_l = M_j^+ (T_k^+ S T_k) M_j \\ &= M_j^+ S_k M_j. \end{aligned} \tag{48}$$

To apply (48) for the purpose of finding the additional matrices S_{F_2} and S_{F_3} required to complete the treatment of the symmetrical Y junction, the matrices F_2 and F_3 must be written in the form of (45); M_k may be either F_1 or R_1. As may be verified by employing Table A,

$$F_1 = R_1^{-1} F_2 R_1 = R_2^{-1} F_3 R_2. \tag{49}$$

IV. Applications

Preferred asymmetry parameters may be tabulated for the several common types of waveguide junctions. Convenient tabulations take the form of pairs of equal matrices, comparison of which, element for element, yields the asymmetry parameters in terms of the conventional scattering parameters, and conversely.

Tables are assigned Roman numerals which correspond to the type of symmetrical waveguide junction considered. Within these principal divisions, according to junction type, each particular symmetry, or combination of symmetries, is distinguished by a letter following the Roman numeral. Due to limitations of space, only those tables required in the body of the paper are given. On the extreme right is a drawing of a common form of the type of waveguide junction considered. This drawing should be examined with care as certain information in respect to circuit conventions essential for the use of the tables is given only in this form. First, the pertinent symmetry is indicated. Second, the waveguide leads of the junction are distinguished by circled Arabic numerals; these numerals correspond to the port designations in the equivalent circuit for the junction. Third, reference or terminal planes are indicated simply by truncating the waveguide leads. The arrows across the terminal planes indicate the assigned polarity.

The tables are divided into two columns. Consider the column on the left designated "Natural Basis." The two matrices in this column are both the conventional (normalized voltage) scattering matrix for the junction S. The upper matrix is essentially the definition of $S = (\mathcal{S}_{ij})$ for the junction. If $\mathcal{S}_{ij} = \mathcal{S}_{ji}$, reciprocity constraints have been imposed. The lower matrix is the scattering matrix written in terms of the preferred parameters. The lower case letters are the asymmetry parameters. The remaining parameters, upper case letters, are symmetry parameters.

Now consider the column on the right designated "Transformed Basis." The matrices in this column are both related to the conventional scattering matrix by the transformation T_M (the subscript M stands for the pertinent symmetry in the particular table), *i.e.*,

$$S_M = T_M^+ S T_M. \tag{50}$$

The upper matrix is S_M written in terms of the preferred parameters, while the lower matrix is S_M written in terms of the elements of the conventional scattering matrix.

Eq. (50) may be given a network interpretation.[3] If one defines a $2N$-port with scattering matrix $S\{T_M\}$,

$$S\{T_M\} = \left[\begin{array}{c|c} 0 & T_M^+ \\ \hline T_M & 0 \end{array}\right], \tag{51}$$

then the N-ports represented by S and S_M are related as shown in Fig. 2. The tandem connection of S with $S\{T_M\}$, in accordance with the terminal markings in Fig. 2, yield a network representation for S_M. Note that each line on the circuit diagram represents a waveguide port or terminal *pair*.

The symmetry and asymmetry parameters have a variety of straightforward applications. These will be

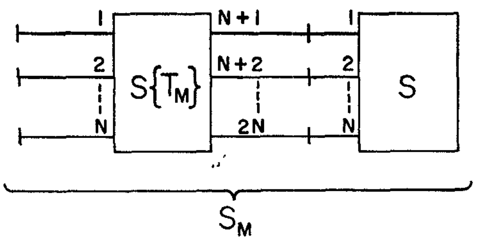


Fig. 2—Network representation of S_M.

illustrated by two examples involving 1) a hybrid-T junction, and 2) a short-slot directional coupler, both frequently encountered in practice.

Consider a hybrid-T junction such as, for example, shown in Table II. The asymmetry parameters for this junction may be determined by measuring the elements of the scattering matrix $S=(s_{ij})$ and then substituting in the second matrix listed in the second column,

$$a_{13} = \frac{1}{2}(s_{11} - s_{22}) = \text{reflection difference},$$

$$a_{14} = \frac{1}{\sqrt{2}}(s_{14} - s_{24}) = H\text{-arm balance depth},$$

$$a_{23} = \frac{1}{\sqrt{2}}(s_{13} - s_{23}) = E\text{-arm balance depth},$$

$$a_{24} = s_{34} \qquad = E\text{-}H \text{ arm isolation}.$$

However, Fig. 2 indicates how these asymmetry parameters might be measured directly provided the network $S\{T_F\}$, *cf.* (51),

$$S\{T_F\} = \frac{1}{\sqrt{2}}\left[\begin{array}{cccc|cccc} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2} \\ \hline 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 & 0 \end{array}\right], \qquad (52)$$

were available. The equivalent circuit of $S\{T_F\}$, shown in Fig. 3, consists of an ideal hybrid-T and two direct connections, as may be verified by inspection. Thus, if a suitable high-quality hybrid-T junction is available, the asymmetry parameters of a second hybrid-T junction may be measured directly by connecting these two as required by the terminal markings for Fig. 2.

The four asymmetry parameters introduced to describe an arbitrary reciprocal hybrid-T junction are all linearly independent. However, if the hybrid-T junction is also lossless, certain nonlinear relations are forced among these parameters and the symmetry parameters of the junction. Some interesting conclusions for *nearly symmetrical*, *nearly matched* hybrid-T junctions may be drawn from a simple perturbation calculation.

The condition that the junction be lossless is that the scattering matrix S or S_F be unitary. Partition the matrix S_F;

$$S_F = \left\{\begin{array}{cc|cc} \mathcal{A}_{11} & \mathcal{A}_{12} & a_{13} & a_{14} \\ \mathcal{A}_{12} & \mathcal{A}_{22} & a_{23} & a_{24} \\ \hline a_{13} & a_{23} & \mathcal{A}_{33} & \mathcal{A}_{34} \\ a_{14} & a_{24} & \mathcal{A}_{34} & \mathcal{A}_{44} \end{array}\right\} = \left(\begin{array}{c|c} \mathcal{A}_{\text{I I}} & a_{\text{I II}} \\ \hline a_{\text{I II}} & \mathcal{A}_{\text{II II}} \end{array}\right), \qquad (53)$$

as shown in (53). Since the junction is *nearly symmetrical*, every element of $a_{\text{I II}}$ is small. Neglecting squares of small quantities, the unitary condition $S_F S_F^+ = I$ yields:

$$\mathcal{A}_{\text{I I}}\, \mathcal{A}_{\text{I I}}^+ = I, \qquad (54a)$$

$$\mathcal{A}_{\text{II II}}\, \mathcal{A}_{\text{II II}}^+ = I; \qquad (54b)$$

$$\tilde{a}_{\text{I II}}\, \mathcal{A}_{\text{I}}^+{}_{\text{I}} + \mathcal{A}_{\text{II II}}\, a_{\text{I}}^+{}_{\text{II}} = 0, \qquad (55a)$$

$$\mathcal{A}_{\text{I I}}\, a_{\text{I}}^*{}_{\text{II}} + a_{\text{I II}}\, \mathcal{A}_{\text{II}}^+{}_{\text{II}} = 0. \qquad (55b)$$

Eqs. (54a) and (54b) state that, to first order, the same relations exist among the symmetry parameters of the hybrid-T junction as would obtain if the junction were perfectly symmetrical. In particular,

$$|\mathcal{A}_{11}|^2 + |\mathcal{A}_{12}|^2 = 1, \qquad |\mathcal{A}_{33}|^2 + |\mathcal{A}_{34}|^2 = 1;$$
$$|\mathcal{A}_{11}| = |\mathcal{A}_{22}|, \qquad |\mathcal{A}_{33}| = |\mathcal{A}_{44}|. \qquad (56)$$

From Table II, column 2, it may be seen that

$$s_{33} = \mathcal{A}_{22} \quad \text{and} \quad s_{44} = \mathcal{A}_{44}, \qquad (57)$$

so that if the hybrid-T junction is *nearly matched*, $\mathcal{A}_{22}$ and $\mathcal{A}_{44}$ are so small that squares of $|\mathcal{A}_{ii}|^2$ may be neglected. (This also implies that, to first order, $|\mathcal{A}_{12}|^2 = |\mathcal{A}_{34}|^2 = 1$.) Eqs. (55a) and (55b) then reduce to

$$\tilde{a}_{\text{I II}}\, U \mathcal{A}_{12}^* + \mathcal{A}_{34} U a_{\text{I}}^+{}_{\text{II}} = 0, \qquad (58a)$$

$$\mathcal{A}_{12} U a_{\text{I}}^*{}_{\text{II}} + a_{\text{I II}} U \mathcal{A}_{34}^* = 0; \qquad (58b)$$

$$U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \qquad (58c)$$

It follows directly from either (58a) or (58b) that

$$|a_{13}| = |a_{24}| \quad \text{and} \quad |a_{14}| = |a_{23}|. \qquad (59)$$

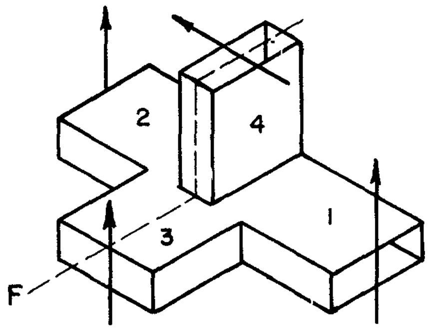
For a second example, consider a junction with many symmetries such as a (short-slot) directional coupler. This junction may be asymmetric in many ways and the analytical asymmetry parameters may aid in the determination of where the symmetry defect lies. To avoid the specialty of a purely numerical example, consider that the scattering matrix of a coupler $S=(s_{ij})$ has been measured and was found to be (see Fig. 4)

$$S = (s_{ij}) = \begin{Bmatrix} \alpha p^2 & \beta p & \gamma p^2 & \delta p \\ \beta p & \alpha & \delta p & \gamma \\ \gamma p^2 & \delta p & \alpha p^2 & \beta p \\ \delta p & \gamma & \beta p & \alpha \end{Bmatrix}. \qquad (60)$$

As compared to the somewhat more symmetrical 4-port junction (see Fig. 5) considered in Table III, the short-slot coupler cannot be expected to possess R_1 or F_3 symmetry. However, it is pertinent to compute the asymmetry parameters associated with F_1, F_2 and R_2.

TABLE II
HYBRID T JUNCTION F SYMMETRY PLANE

$$F=\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix}\qquad T=\frac{1}{\sqrt{2}}\begin{bmatrix}1&0&1&0\\1&0&-1&0\\0&\sqrt{2}&0&0\\0&0&0&\sqrt{2}\end{bmatrix}$$



Natural Basis	$S=\begin{bmatrix}\mathcal{S}_{11}&\mathcal{S}_{12}&\mathcal{S}_{13}&\mathcal{S}_{14}\\\mathcal{S}_{12}&\mathcal{S}_{22}&\mathcal{S}_{23}&\mathcal{S}_{24}\\\mathcal{S}_{13}&\mathcal{S}_{23}&\mathcal{S}_{33}&\mathcal{S}_{34}\\\mathcal{S}_{14}&\mathcal{S}_{24}&\mathcal{S}_{34}&\mathcal{S}_{44}\end{bmatrix}$	$S=\frac{1}{2}\begin{bmatrix}(\mathcal{A}_{11}+\mathcal{A}_{33}+2a_{13})&(\mathcal{A}_{11}-\mathcal{A}_{33})&\sqrt{2}(\mathcal{A}_{12}+a_{23})&\sqrt{2}(\mathcal{A}_{34}+a_{14})\\(\mathcal{A}_{11}-\mathcal{A}_{33})&(\mathcal{A}_{11}+\mathcal{A}_{33}-2a_{13})&\sqrt{2}(\mathcal{A}_{12}-a_{23})&-\sqrt{2}(\mathcal{A}_{34}-a_{14})\\\sqrt{2}(\mathcal{A}_{12}+a_{23})&\sqrt{2}(\mathcal{A}_{12}-a_{23})&2\mathcal{A}_{22}&2a_{24}\\\sqrt{2}(\mathcal{A}_{34}+a_{14})&-\sqrt{2}(\mathcal{A}_{34}-a_{14})&2a_{24}&2\mathcal{A}_{44}\end{bmatrix}$
Transformed Basis	$S_F=\begin{bmatrix}\mathcal{A}_{11}&\mathcal{A}_{12}&a_{13}&a_{14}\\\mathcal{A}_{12}&\mathcal{A}_{22}&a_{23}&a_{24}\\a_{13}&a_{23}&\mathcal{A}_{33}&\mathcal{A}_{34}\\a_{14}&a_{24}&\mathcal{A}_{34}&\mathcal{A}_{44}\end{bmatrix}$	$S_F=\frac{1}{2}\begin{bmatrix}(\mathcal{S}_{11}+\mathcal{S}_{22}+2\mathcal{S}_{12})&\sqrt{2}(\mathcal{S}_{13}+\mathcal{S}_{23})&(\mathcal{S}_{11}-\mathcal{S}_{22})&\sqrt{2}(\mathcal{S}_{14}+\mathcal{S}_{24})\\\sqrt{2}(\mathcal{S}_{13}+\mathcal{S}_{23})&2\mathcal{S}_{33}&\sqrt{2}(\mathcal{S}_{13}-\mathcal{S}_{23})&2\mathcal{S}_{34}\\(\mathcal{S}_{11}-\mathcal{S}_{22})&\sqrt{2}(\mathcal{S}_{13}-\mathcal{S}_{23})&(\mathcal{S}_{11}+\mathcal{S}_{22}-2\mathcal{S}_{12})&\sqrt{2}(\mathcal{S}_{14}-\mathcal{S}_{24})\\\sqrt{2}(\mathcal{S}_{14}+\mathcal{S}_{24})&2\mathcal{S}_{34}&\sqrt{2}(\mathcal{S}_{14}-\mathcal{S}_{24})&2\mathcal{S}_{44}\end{bmatrix}$

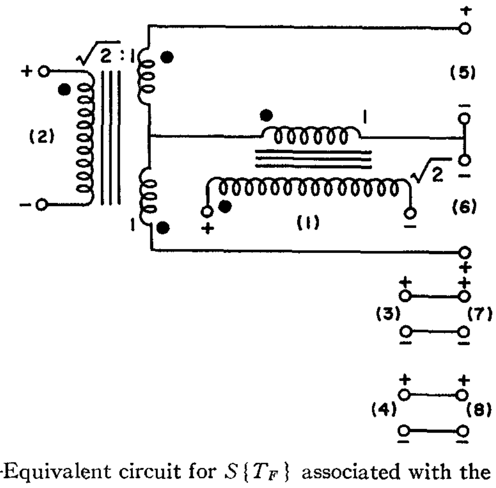


Fig. 3—Equivalent circuit for $S\{T_F\}$ associated with the hybrid-T.

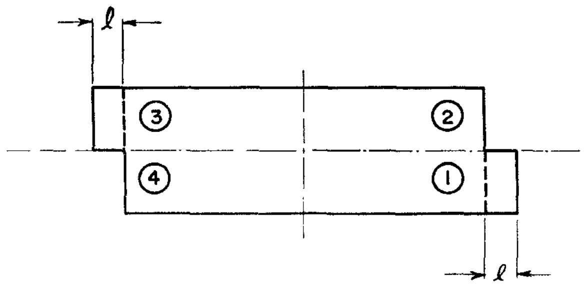


Fig. 4—Symmetry of the coupler described by (60).

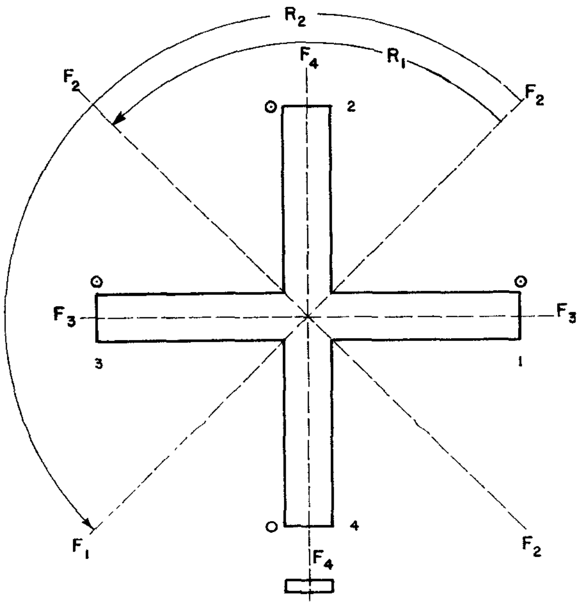
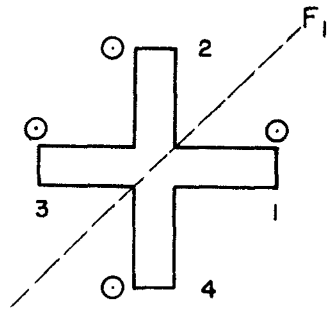


Fig. 5—Symmetrical reciprocal H-plane four-port junction. F_1, symmetry plane; F_2, symmetry plane; F_3, symmetry plane; F_4, symmetry plane; R_1, rotational symmetry; F_1 and R_1 symmetry (implies all remaining operations in group); R_2, rotational symmetry (applicable to short-slot coupler); F_1 and R_2, symmetry (implies all remaining operations in subgroup for short-slot coupler geometry).

TABLE III

(a) Symmetrical H-Plane Four-Port Junction F_1 Symmetry Plane

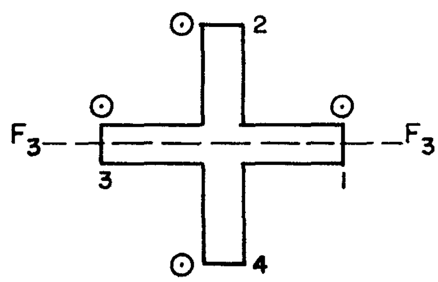
$$F_1=\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\qquad T_{F_1}=\frac{1}{\sqrt{2}}\begin{bmatrix}1&0&1&0\\1&0&-1&0\\0&1&0&1\\0&1&0&-1\end{bmatrix}$$



Natural Basis	$S=\begin{bmatrix}\mathcal{S}_{11}&\mathcal{S}_{12}&\mathcal{S}_{13}&\mathcal{S}_{14}\\\mathcal{S}_{12}&\mathcal{S}_{22}&\mathcal{S}_{23}&\mathcal{S}_{24}\\\mathcal{S}_{13}&\mathcal{S}_{23}&\mathcal{S}_{33}&\mathcal{S}_{34}\\\mathcal{S}_{14}&\mathcal{S}_{24}&\mathcal{S}_{34}&\mathcal{S}_{44}\end{bmatrix}$	$S=\frac{1}{2}\begin{bmatrix}(\mathcal{A}_{11}+\mathcal{A}_{33}+2a_{13})&(\mathcal{A}_{11}-\mathcal{A}_{33})&(\mathcal{A}_{12}+\mathcal{A}_{34}+a_{14}+a_{23})&(\mathcal{A}_{12}-\mathcal{A}_{34}-a_{14}+a_{23})\\(\mathcal{A}_{11}-\mathcal{A}_{33})&(\mathcal{A}_{11}+\mathcal{A}_{33}-2a_{13})&(\mathcal{A}_{12}-\mathcal{A}_{34}+a_{14}-a_{23})&(\mathcal{A}_{12}+\mathcal{A}_{34}-a_{14}-a_{23})\\(\mathcal{A}_{12}+\mathcal{A}_{34}+a_{14}+a_{23})&(\mathcal{A}_{12}-\mathcal{A}_{34}+a_{14}-a_{23})&(\mathcal{A}_{22}+\mathcal{A}_{44}+2a_{14})&(\mathcal{A}_{22}-\mathcal{A}_{44})\\(\mathcal{A}_{12}-\mathcal{A}_{34}-a_{14}+a_{23})&(\mathcal{A}_{12}+\mathcal{A}_{34}-a_{14}-a_{23})&(\mathcal{A}_{22}-\mathcal{A}_{44})&(\mathcal{A}_{22}+\mathcal{A}_{44}-2a_{24})\end{bmatrix}$
Transformed Basis	$S_{F_1}=\begin{bmatrix}\mathcal{A}_{11}&\mathcal{A}_{12}&a_{13}&a_{14}\\\mathcal{A}_{12}&\mathcal{A}_{22}&a_{23}&a_{24}\\a_{13}&a_{23}&\mathcal{A}_{33}&\mathcal{A}_{34}\\a_{14}&a_{24}&\mathcal{A}_{34}&\mathcal{A}_{44}\end{bmatrix}$	$S_{F_1}=\frac{1}{2}\begin{bmatrix}(\mathcal{S}_{11}+\mathcal{S}_{22}+2\mathcal{S}_{12})&(\mathcal{S}_{13}+\mathcal{S}_{14}+\mathcal{S}_{23}+\mathcal{S}_{24})&(\mathcal{S}_{11}-\mathcal{S}_{22})&(\mathcal{S}_{13}-\mathcal{S}_{14}+\mathcal{S}_{23}-\mathcal{S}_{24})\\(\mathcal{S}_{13}+\mathcal{S}_{14}+\mathcal{S}_{23}+\mathcal{S}_{24})&(\mathcal{S}_{33}+\mathcal{S}_{44}+2\mathcal{S}_{34})&(\mathcal{S}_{13}-\mathcal{S}_{23}+\mathcal{S}_{14}-\mathcal{S}_{24})&(\mathcal{S}_{33}-\mathcal{S}_{44})\\(\mathcal{S}_{11}-\mathcal{S}_{22})&(\mathcal{S}_{13}+\mathcal{S}_{14}-\mathcal{S}_{23}-\mathcal{S}_{24})&(\mathcal{S}_{11}+\mathcal{S}_{22}-2\mathcal{S}_{12})&(\mathcal{S}_{13}-\mathcal{S}_{14}-\mathcal{S}_{23}+\mathcal{S}_{24})\\(\mathcal{S}_{13}+\mathcal{S}_{23}-\mathcal{S}_{14}-\mathcal{S}_{24})&(\mathcal{S}_{33}-\mathcal{S}_{44})&(\mathcal{S}_{13}-\mathcal{S}_{23}-\mathcal{S}_{14}+\mathcal{S}_{24})&(\mathcal{S}_{33}+\mathcal{S}_{44}-2\mathcal{S}_{34})\end{bmatrix}$

(b) Symmetrical H-Plane Four-Port Junction F_3 Symmetry Plane

$$F_3=\begin{bmatrix}1&0&0&0\\0&0&0&1\\0&0&1&0\\0&1&0&0\end{bmatrix}\qquad T_{F_3}=\frac{1}{\sqrt{2}}\begin{bmatrix}\sqrt{2}&0&0&0\\0&0&1&1\\0&\sqrt{2}&0&0\\0&0&1&-1\end{bmatrix}$$

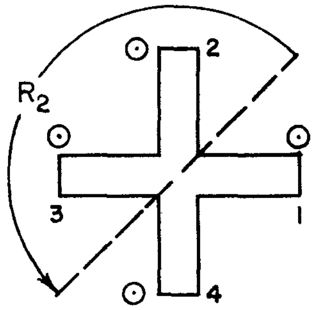


Natural Basis	$S=\begin{bmatrix}\mathcal{S}_{11}&\mathcal{S}_{12}&\mathcal{S}_{13}&\mathcal{S}_{14}\\\mathcal{S}_{12}&\mathcal{S}_{22}&\mathcal{S}_{23}&\mathcal{S}_{24}\\\mathcal{S}_{13}&\mathcal{S}_{23}&\mathcal{S}_{33}&\mathcal{S}_{34}\\\mathcal{S}_{14}&\mathcal{S}_{24}&\mathcal{S}_{34}&\mathcal{S}_{44}\end{bmatrix}$	$S=\frac{1}{2}\begin{bmatrix}2\mathcal{C}_{11}&\sqrt{2}(\mathcal{C}_{13}+c_{14})&2\mathcal{C}_{12}&\sqrt{2}(\mathcal{C}_{13}-c_{14})\\\sqrt{2}(\mathcal{C}_{13}+c_{14})&(\mathcal{C}_{33}+\mathcal{C}_{44}+2c_{34})&\sqrt{2}(\mathcal{C}_{23}+c_{24})&(\mathcal{C}_{33}-\mathcal{C}_{44})\\2\mathcal{C}_{12}&\sqrt{2}(\mathcal{C}_{23}+c_{24})&2\mathcal{C}_{22}&\sqrt{2}(\mathcal{C}_{23}-c_{24})\\\sqrt{2}(\mathcal{C}_{13}-c_{14})&(\mathcal{C}_{33}-\mathcal{C}_{44})&\sqrt{2}(\mathcal{C}_{23}-c_{24})&(\mathcal{C}_{33}+\mathcal{C}_{44}-2\mathcal{C}_{34})\end{bmatrix}$
Transformed Basis	$S_{F_3}=\begin{bmatrix}\mathcal{C}_{11}&\mathcal{C}_{12}&\mathcal{C}_{13}&c_{14}\\\mathcal{C}_{12}&\mathcal{C}_{22}&\mathcal{C}_{23}&c_{24}\\\mathcal{C}_{13}&\mathcal{C}_{23}&\mathcal{C}_{33}&c_{34}\\c_{14}&c_{24}&c_{34}&\mathcal{C}_{44}\end{bmatrix}$	$S_{F_3}=\frac{1}{2}\begin{bmatrix}2\mathcal{S}_{11}&2\mathcal{S}_{13}&\sqrt{2}(\mathcal{S}_{12}+\mathcal{S}_{14})&\sqrt{2}(\mathcal{S}_{12}-\mathcal{S}_{14})\\2\mathcal{S}_{13}&2\mathcal{S}_{33}&\sqrt{2}(\mathcal{S}_{23}+\mathcal{S}_{34})&\sqrt{2}(\mathcal{S}_{23}-\mathcal{S}_{34})\\\sqrt{2}(\mathcal{S}_{12}+\mathcal{S}_{14})&\sqrt{2}(\mathcal{S}_{23}+\mathcal{S}_{34})&(\mathcal{S}_{22}+\mathcal{S}_{44}+2\mathcal{S}_{24})&(\mathcal{S}_{22}-\mathcal{S}_{44})\\\sqrt{2}(\mathcal{S}_{12}-\mathcal{S}_{14})&\sqrt{2}(\mathcal{S}_{23}-\mathcal{S}_{34})&(\mathcal{S}_{22}-\mathcal{S}_{44})&(\mathcal{S}_{22}+\mathcal{S}_{44}-2\mathcal{S}_{24})\end{bmatrix}$

TABLE III

(c) SYMMETRICAL RECIPROCAL H-PLANE FOUR-PORT JUNCTION R_2 ROTATIONAL SYMMETRY (CONT'D)

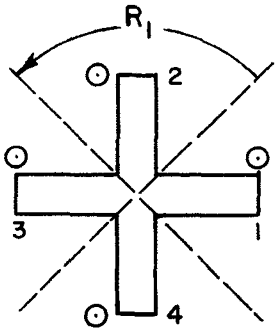
$$R_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad T_{R_2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$



Natural Basis	$S = \begin{bmatrix} \mathcal{S}_{11} & \mathcal{S}_{12} & \mathcal{S}_{13} & \mathcal{S}_{14} \\ \mathcal{S}_{12} & \mathcal{S}_{22} & \mathcal{S}_{23} & \mathcal{S}_{24} \\ \mathcal{S}_{13} & \mathcal{S}_{23} & \mathcal{S}_{33} & \mathcal{S}_{34} \\ \mathcal{S}_{14} & \mathcal{S}_{24} & \mathcal{S}_{34} & \mathcal{S}_{44} \end{bmatrix}$	$S = \frac{1}{2} \begin{bmatrix} (\mathcal{G}_{11}+\mathcal{G}_{33}-2g_{13}) & (\mathcal{G}_{12}-\mathcal{G}_{34}+g_{14}-g_{23}) & (\mathcal{G}_{11}-\mathcal{G}_{33}) & (\mathcal{G}_{12}+\mathcal{G}_{34}-g_{14}-g_{23}) \\ (\mathcal{G}_{12}-\mathcal{G}_{34}+g_{14}-g_{23}) & (\mathcal{G}_{22}+\mathcal{G}_{44}+2g_{24}) & (\mathcal{G}_{12}+\mathcal{G}_{34}+g_{14}+g_{23}) & (\mathcal{G}_{22}-\mathcal{G}_{44}) \\ (\mathcal{G}_{11}-\mathcal{G}_{33}) & (\mathcal{G}_{12}+\mathcal{G}_{34}+g_{14}+g_{23}) & (\mathcal{G}_{11}+\mathcal{G}_{33}+2g_{13}) & (\mathcal{G}_{12}-\mathcal{G}_{34}-g_{14}+g_{23}) \\ (\mathcal{G}_{12}+\mathcal{G}_{34}-g_{14}-g_{23}) & (\mathcal{G}_{22}-\mathcal{G}_{44}) & (\mathcal{G}_{12}-\mathcal{G}_{34}-g_{14}+g_{23}) & (\mathcal{G}_{22}+\mathcal{G}_{44}-2g_{24}) \end{bmatrix}$
Transformed Basis	$S_{R_2} = \begin{bmatrix} \mathcal{G}_{11} & \mathcal{G}_{12} & g_{13} & g_{14} \\ \mathcal{G}_{12} & \mathcal{G}_{22} & g_{23} & g_{24} \\ g_{13} & g_{23} & \mathcal{G}_{33} & \mathcal{G}_{34} \\ g_{14} & g_{24} & \mathcal{G}_{34} & \mathcal{G}_{44} \end{bmatrix}$	$S_{R_2} = \frac{1}{2} \begin{bmatrix} (\mathcal{S}_{11}+\mathcal{S}_{33}+2\mathcal{S}_{13}) & (\mathcal{S}_{12}+\mathcal{S}_{14}+\mathcal{S}_{23}+\mathcal{S}_{34}) & (\mathcal{S}_{33}-\mathcal{S}_{11}) & (\mathcal{S}_{12}-\mathcal{S}_{14}+\mathcal{S}_{23}-\mathcal{S}_{34}) \\ (\mathcal{S}_{12}+\mathcal{S}_{23}+\mathcal{S}_{14}+\mathcal{S}_{34}) & (\mathcal{S}_{22}+\mathcal{S}_{44}+2\mathcal{S}_{24}) & (\mathcal{S}_{23}-\mathcal{S}_{12}-\mathcal{S}_{14}+\mathcal{S}_{34}) & (\mathcal{S}_{22}-\mathcal{S}_{44}) \\ (\mathcal{S}_{33}-\mathcal{S}_{11}) & (\mathcal{S}_{23}-\mathcal{S}_{14}-\mathcal{S}_{12}+\mathcal{S}_{34}) & (\mathcal{S}_{11}+\mathcal{S}_{33}-2\mathcal{S}_{13}) & (\mathcal{S}_{14}-\mathcal{S}_{12}) \\ (\mathcal{S}_{12}-\mathcal{S}_{14}+\mathcal{S}_{23}-\mathcal{S}_{34}) & (\mathcal{S}_{22}-\mathcal{S}_{44}) & (\mathcal{S}_{14}-\mathcal{S}_{12}) & (\mathcal{S}_{22}+\mathcal{S}_{44}-2\mathcal{S}_{24}) \end{bmatrix}$

(d) SYMMETRICAL H-PLANE FOUR-PORT JUNCTION R_1 ROTATIONAL SYMMETRY

$$R_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad T_{R_1} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$



Natural Basis	$S = \begin{bmatrix} \mathcal{S}_{11} & \mathcal{S}_{12} & \mathcal{S}_{13} & \mathcal{S}_{14} \\ \mathcal{S}_{12} & \mathcal{S}_{22} & \mathcal{S}_{23} & \mathcal{S}_{24} \\ \mathcal{S}_{13} & \mathcal{S}_{23} & \mathcal{S}_{33} & \mathcal{S}_{34} \\ \mathcal{S}_{14} & \mathcal{S}_{24} & \mathcal{S}_{34} & \mathcal{S}_{44} \end{bmatrix}$	$\begin{aligned} \mathcal{S}_{11} &= \tfrac{1}{4}[\mathcal{E}_{11} + \mathcal{E}_{33} + e_{24} + e_{42} + 2(e_{12} + e_{13} + e_{14} + e_{23} + e_{32}) + 2\mathcal{E}_{22}] \\ \mathcal{S}_{12} &= \tfrac{1}{4}[\mathcal{E}_{11} - \mathcal{E}_{33} - j(e_{24} - e_{42}) + (1+j)(e_{12} - e_{23}) + (1-j)(e_{14} - e_{32})] \\ \mathcal{S}_{13} &= \tfrac{1}{4}[\mathcal{E}_{11} + \mathcal{E}_{33} - (e_{24} + e_{42}) + 2e_{13} - 2\mathcal{E}_{22}] \\ \mathcal{S}_{14} &= \tfrac{1}{4}[\mathcal{E}_{11} - \mathcal{E}_{33} - j(e_{42} - e_{24}) + (1+j)(e_{14} - e_{32}) + (1-j)(e_{12} - e_{23})] \\ \mathcal{S}_{22} &= \tfrac{1}{4}[\mathcal{E}_{11} + \mathcal{E}_{33} - (e_{24} + e_{42}) - j2(e_{14} - e_{12} + e_{32} - e_{23}) - 2e_{13} + 2\mathcal{E}_{22}] \\ \mathcal{S}_{23} &= \tfrac{1}{4}[\mathcal{E}_{11} - \mathcal{E}_{33} - j(e_{42} - e_{24}) + (1+j)(e_{32} - e_{14}) + (1-j)(e_{23} - e_{12})] \\ \mathcal{S}_{24} &= \tfrac{1}{4}[\mathcal{E}_{11} + \mathcal{E}_{33} + e_{24} + e_{42} - 2e_{14} - 2\mathcal{E}_{22}] \\ \mathcal{S}_{33} &= \tfrac{1}{4}[\mathcal{E}_{11} + \mathcal{E}_{33} + e_{24} + e_{42} + 2(e_{13} - e_{12} - e_{14} - e_{23} - e_{32}) + 2\mathcal{E}_{22}] \\ \mathcal{S}_{34} &= \tfrac{1}{4}[\mathcal{E}_{11} - \mathcal{E}_{33} - j(e_{24} - e_{42}) + (1+j)(e_{23} - e_{12}) + (1-j)(e_{32} - e_{12})] \\ \mathcal{S}_{44} &= \tfrac{1}{4}[\mathcal{E}_{11} + \mathcal{E}_{33} - (e_{24} + e_{42}) - j2(e_{12} - e_{14} + e_{23} - e_{32}) - 2e_{13} + 2\mathcal{E}_{22}] \end{aligned}$
Transformed Basis	$S_{R_1} = \begin{bmatrix} \mathcal{E}_{11} & e_{12} & e_{13} & e_{14} \\ e_{12} & \mathcal{E}_{22} & e_{23} & e_{24} \\ e_{13} & e_{23} & \mathcal{E}_{33} & e_{34} \\ e_{14} & e_{24} & e_{34} & \mathcal{E}_{22} \end{bmatrix}$	$\begin{aligned} \mathcal{E}_{11} &= \tfrac{1}{4}[\mathcal{S}_{11} + \mathcal{S}_{22} + \mathcal{S}_{33} + \mathcal{S}_{44} + 2(\mathcal{S}_{12} + \mathcal{S}_{13} + \mathcal{S}_{14} + \mathcal{S}_{23} + \mathcal{S}_{24} + \mathcal{S}_{34})] \\ e_{12} &= \tfrac{1}{4}[\mathcal{S}_{11} - \mathcal{S}_{33} - j(\mathcal{S}_{22} - \mathcal{S}_{44}) + (1-j)(\mathcal{S}_{12} - \mathcal{S}_{34}) + (1+j)(\mathcal{S}_{14} - \mathcal{S}_{23})] \\ e_{13} &= \tfrac{1}{4}[\mathcal{S}_{11} + \mathcal{S}_{33} - (\mathcal{S}_{22} + \mathcal{S}_{44}) + 2(\mathcal{S}_{13} - \mathcal{S}_{24})] \\ e_{14} &= \tfrac{1}{4}[\mathcal{S}_{11} - \mathcal{S}_{33} - j(\mathcal{S}_{44} - \mathcal{S}_{22}) + (1+j)(\mathcal{S}_{12} - \mathcal{S}_{34}) + (1-j)(\mathcal{S}_{14} - \mathcal{S}_{23})] \\ \mathcal{E}_{22} &= \tfrac{1}{4}[\mathcal{S}_{11} + \mathcal{S}_{22} + \mathcal{S}_{33} + \mathcal{S}_{44} - 2(\mathcal{S}_{13} + \mathcal{S}_{24})] \\ e_{23} &= \tfrac{1}{4}[\mathcal{S}_{11} - \mathcal{S}_{33} - j(\mathcal{S}_{22} - \mathcal{S}_{44}) + (1+j)(\mathcal{S}_{23} - \mathcal{S}_{14}) + (1-j)(\mathcal{S}_{34} - \mathcal{S}_{12})] \\ e_{24} &= \tfrac{1}{4}[\mathcal{S}_{11} + \mathcal{S}_{33} - (\mathcal{S}_{22} + \mathcal{S}_{44}) - j2(\mathcal{S}_{14} - \mathcal{S}_{34} - \mathcal{S}_{12} + \mathcal{S}_{23}) + 2(\mathcal{S}_{24} - \mathcal{S}_{13})] \\ e_{32} &= \tfrac{1}{4}[\mathcal{S}_{11} - \mathcal{S}_{33} - j(\mathcal{S}_{44} - \mathcal{S}_{22}) + (1+j)(\mathcal{S}_{34} - \mathcal{S}_{12}) + (1-j)(\mathcal{S}_{23} - \mathcal{S}_{14})] \\ \mathcal{E}_{33} &= \tfrac{1}{4}[\mathcal{S}_{11} + \mathcal{S}_{22} + \mathcal{S}_{33} + \mathcal{S}_{44} + 2(\mathcal{S}_{13} - \mathcal{S}_{12} - \mathcal{S}_{14} - \mathcal{S}_{23} + \mathcal{S}_{24} - \mathcal{S}_{34})] \\ e_{42} &= \tfrac{1}{4}[\mathcal{S}_{11} + \mathcal{S}_{33} - (\mathcal{S}_{22} + \mathcal{S}_{44}) - j2(\mathcal{S}_{12} - \mathcal{S}_{23} - \mathcal{S}_{14} + \mathcal{S}_{34}) + 2(\mathcal{S}_{24} - \mathcal{S}_{13})] \end{aligned}$

Asymmetry parameters associated with F_1, Table III(a):

$$\begin{aligned} a_{13} &= \tfrac{1}{2}\alpha(p^2 - 1) \\ a_{14} &= \tfrac{1}{2}\gamma(p^2 + 1) \\ a_{23} &= \tfrac{1}{2}\gamma(p^2 - 1) \\ a_{24} &= \tfrac{1}{2}\alpha(p^2 - 1). \end{aligned} \tag{61}$$

Asymmetry parameters associated with F_2:

$$\begin{aligned} b_{13} &= \tfrac{1}{2}\alpha(p^2 - 1) \\ b_{14} &= -\tfrac{1}{2}\gamma(p^2 - 1) \\ b_{23} &= \tfrac{1}{2}\gamma(p^2 - 1) \\ b_{24} &= -\tfrac{1}{2}\alpha(p^2 - 1). \end{aligned} \tag{62}$$

Asymmetry parameters associated with R_2, Table III(c):

$$g_{13} = g_{23} = g_{14} = g_{24} = 0. \tag{63}$$

Thus, the coupler, the scattering matrix of which had the form (60), has a symmetry (or asymmetry) equivalent to that shown in Fig. 4. For example, if

$$p = \exp\left(-j2\pi \frac{l}{\lambda_g}\right), \tag{64}$$

then the matrix (60) corresponds to that of a coupler which is perfectly symmetrical *except for lengths of guide* l indicated in Fig. 4. The implications as regard dimensional checks or compensating cuts to be made on the component are evident.

Orthogonality Relationships for Waveguides and Cavities with Inhomogeneous Anisotropic Media*

ALFRED T. VILLENEUVE†

Summary—**A modified reciprocity theorem forms the basis of development of orthogonality relationships for modes in waveguides and in cavities containing inhomogeneous, anisotropic media. In the lossless case certain of these may be interpreted in terms of power flow and energy storage. The special case of magnetized gyrotropic media is discussed for longitudinal and transverse magnetization.**

Introduction

RECENTLY the use of anisotropic materials has been the subject of numerous theoretical and experimental investigations.[1] Such materials are characterized in their macroscopic behavior by tensor permittivities or permeabilities. When these tensors are unsymmetric, the media may be termed "nonreciprocal" since the usual reciprocity theorem[2] does not apply to them. This nonreciprocal behavior finds applications in such devices as circulators, gyrators, load isolators and nonreciprocal phase shifters.[3]

One important special class of nonreciprocal media is that known as gyrotropic media, wherein application of a dc magnetic field causes the permittivity or permeability (hereafter referred to as constitutive parameters) to become an unsymmetric tensor. Two examples are gaseous plasma and ferromagnetic materials, especially low loss, magnetically-saturated ferrites.

Although the usual reciprocity theorem is not valid, a modified reciprocity theorem[4] does apply to anisotropic media. In this theorem, media characterized by transposed tensor constitutive parameters are employed in addition to the original media. In this paper, the modified reciprocity theorem forms a basis for the derivation of orthogonality relationships for modes in waveguides and cavities containing inhomogeneous, anisotropic media.

Let us denote the general form of the constitutive parameters in orthogonal coordinate systems as

$$[\epsilon] = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \hat{\epsilon}_{12} & \epsilon_{22} & \epsilon_{23} \\ \hat{\epsilon}_{13} & \hat{\epsilon}_{23} & \epsilon_{33} \end{bmatrix} \quad [\mu] = \begin{bmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \hat{\mu}_{12} & \mu_{22} & \mu_{23} \\ \hat{\mu}_{13} & \hat{\mu}_{23} & \mu_{33} \end{bmatrix}. \tag{1}$$

In this notation the careted symbols, $\hat{\epsilon}_{ij}$ and $\hat{\mu}_{ij}$, are the elements in the ith row and jth column of the constitutive parameter tensors for media characterized by the transposes of the above tensors. These media shall be referred to as "transposed media." In the case of gyrotropic media this has physical significance, since revers-

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[1] A complete list of references is impractical here and any attempt at making specific references would be difficult. For extensive lists of references the reader is referred to Proc. IRE, vol. 44, pp. 1229–1516; October, 1956.

[2] S. A. Schelkunoff, "Electromagnetic Waves," D. Van Nostrand Co., Inc., New York, N. Y., 1st ed., p. 478; 1943.

[3] C. L. Hogan, "The elements of non-reciprocal microwave devices," Proc. IRE, vol. 44, pp. 1345–1368; October, 1956.

[4] R. F. Harrington and A. T. Villeneuve, "Reciprocity relationships for gyrotropic media," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, pp. 308–310; July, 1958.